Kalman Tracking Filter in 3-D Space

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Abstract: *In this work a Kalman filter for tracking Unmanned Aerial Vehicle (UAV) moving near the base lane of a bistatic radar system and illuminated by GPS signals in three-dimensional (3-D) space is suggested. Continuous-time and discrete-time state equations are derived to define a linear time-invariant model. Model matrices describing the behavior of the object, state vectors, measurement vectors and vector noise process with its statistical characteristic are defined. Structures of the discrete state transition matrix of the dynamic model and covariance noise matrix are given. Linear time-invariant model matrices are derived. State model matrices with constant entries, measurement model matrix, state transition matrix of the dynamic model, covariance noise state matrix and covariance matrix of the measurements, are defined. Recurrent Kalman equations are described. To verify a 3-D Kalman tracking a numerical experiment is performed.*

1. Introduction

The requirements to modern guiding systems are becoming more severe. The guiding system has to be able to produce an accurate and fast response despite variations in speed and altitude of the airframe and large parametric uncertainty. To meet these requirements adaptive control techniques are applied. In particular, in case of UAV moving near the base lane of a bistatic radar system and illuminated by L_1 or L_2 signals of GPS transmitters the states of the object can be considered as a behavior of linear and nonlinear dynamic systems. An effective tool to describe dynamic systems is the extended versions of the Kalman filter. It is a powerful technique estimating the states of dynamic systems. Implementation of a nonlinear guidance and control technique based on Kalman filter to track a target in 3D environment attracts much attention [1-6]. For instance, cooperative active target tracking for heterogeneous robots with application to gait monitoring and nonlinear guidance and control laws for 3-D target tracking are discussed in [3, 4].

3D target tracking subject to measurement of uncertainties and 3D model-based tracking for 3-D localization are described in [5, 6]. A drift eliminated attitude and position estimation algorithm in 3D space and flight test evaluation of GPS/INS sensor fusion algorithms for attitude estimation are described in [7, 8]. An extended Kalman filter is used in UAV attitude, heading, and wind estimation based on measurements of GPS/INS and an air data system [9]. Evaluation of matrix square root operations for unscented Kalman filter within a UAV GPS/INS sensor fusion application is given in [10]. Evaluation of approaches to sensor failure

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detection and accommodation for the airspeed sensor on a small UAV is presented in [11]. Performance evaluation of neural network based approaches for airspeed sensor failure accommodation on a small UAV is discussed in [12]. In-depth discussion of the intricacies of different nonlinear Kalman filters as the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) for practical state estimation applications is presented in [13].

Design and implementation of manual remote control system that includes two control systems: automatic control and manual control for UAV is described in [14]. Methodology for real-time control of UAV in the absence of a priori knowledge of the location in an inhospitable flight territory is presented in [15].

Discrete-time state space models, linear state space estimation equations, and demonstration of two dimensional continuous Wiener process acceleration (2-D CWPA)-model are presented in [16].

In the present work a 3-D linear time-invariant dynamic model of the state of a moving UAV target illuminated by L_1 or L_2 signals of GPS transmitters and Kalman tracking algorithm are derived. The numerical results are obtained based on the modified Matlab source code in [16].

The remainder of the paper is organized as follows. In Section II a linear time-invariant model and Pade expansion of the state transition matrix are described. In Section III linear time-invariant model matrices, state vector, power spectral density matrix, model matrices of constant entries, which characterize the behavior of the target, measurement model matrix, dynamic model, and covariance noise state matrix, are defined. In Section IV recurrent Kalman equations are defined. In Section V results of a numerical experiment, validating the performance of the 3-D tracking algorithm, are given. In section VI conclusions are derived.

2. Linear Time-Invariant Model Description

Consider linear time-invariant models described with continuous-time state equations in the form

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}\mathbf{x}(t) + \mathbf{L}\mathbf{w}(t) \tag{1}$$

where $\mathbf{F}$ and $\mathbf{L}$ are model matrices of constant entries characterizing the behavior of the object; $\mathbf{w}(t)$ is a white noise process with a power spectral density $\mathbf{Q}_c$; the initial state vector $\mathbf{x}(0)$ has normal distribution with mean $\mathbf{m}(0)$ and covariance $\mathbf{P}(0)$. A linear time-invariant system defined by equation (1) has Markov-property, which means that the state $\mathbf{x}_k$ given $\mathbf{x}_{k-1}$ is independent from the history of states and measurements.

Digital implementations of Kalman filter require the model (1) to be presented in a discrete form. Whereas the state space model used in the paper is a linear model the discrete state and measurement equations can be written by the expressions

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{v}_{k-1} \tag{2}$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{n}_k \tag{3}$$

where $\mathbf{x}_k$ is the state vector of the system on the time moment k; $\mathbf{y}_k$ is the measurement vector on the time moment k; $\mathbf{v}_{k-1} \sim \mathbf{N}(\mathbf{0}, \mathbf{Q}_{k-1})$ is the zero mean process noise with covariance matrix $\mathbf{Q}_{k-1}$ on the time moment k - 1; $\mathbf{n}_k \sim \mathbf{N}(\mathbf{0}, \mathbf{R}_k)$ is the zero mean measurement noise with

covariance matrix $\mathbf{R}_k$ on the time step; $\mathbf{A}_{k-1}$ is the state transition matrix of the dynamic model; $\mathbf{H}_k$ is the measurement model matrix which maps the true state space into the observed space.
The prior distribution for the state is $\mathbf{x}_0 \sim \mathbf{N}(\mathbf{m}_0, \mathbf{P}_0)$ where parameters $\mathbf{m}_0$ and $\mathbf{P}_0$ are set using the information known about the system under the consideration.
The solution for the state transition matrix $\mathbf{A}_k$ and covariance matrix $\mathbf{Q}_k$ in a discrete form can be given as [16]

$$\mathbf{A}_k = \exp(\mathbf{F}\Delta t_k) \tag{4}$$

$$\mathbf{Q}_k = \mathbf{C}_k \mathbf{D}_k^{-1} . \tag{5}$$

The Pade expansion of the state transition matrix has the form

$$\exp(\mathbf{F}\Delta t_k) = \sum_{n=0} \mathbf{F}^n \frac{(\Delta t_k)^n}{n!} \text{, where } \mathbf{F}^0 = \mathbf{I} . \tag{6}$$

The matrices $\mathbf{C}_k$ and $\mathbf{D}_k$ can be calculated analytically using the following matrix fraction decomposition [16]

$$\begin{pmatrix} \mathbf{C}_k \\ \mathbf{D}_k \end{pmatrix} = \exp\left\{ \begin{pmatrix} \mathbf{F} & \mathbf{L}\mathbf{Q}_c\mathbf{L}^T \\ \mathbf{0} & -\mathbf{F}^T \end{pmatrix} \Delta t_k \right\} \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix}, \tag{7}$$

where

$\Delta t_k = t_{k+1} - t_k$ is the sampling time interval; $\mathbf{0}$ and $\mathbf{I}$ are zero and identity matrices, respectively.

3. Linear Time-Invariant Model Matrices

Consider an object, UAV, illuminated by L_1 or L_2 signals of three GPS transceivers. The object is tracked during its movement in three-dimensional space, based on measurements of reflected GPS signals by modified GPS receivers. It gives measurement data to define the target's position in Cartesian coordinates. The state vector of the target consists of coordinates of the target position $[\,x(t_k)\ y(t_k)\ z(t_k)\,]^T$, velocity $[\,\dot{x}(t_k)\ \dot{y}(t_k)\ \dot{z}(t_k)\,]^T$ and acceleration $[\,\ddot{x}(t_k)\ \ddot{y}(t_k)\ \ddot{z}(t_k)\,]^T$ toward three coordinate axes. In other words, the state vector of a moving target on k-th time moment, $\mathbf{x}(t_k)$ can be expressed as a matrix column from nine elements as follows

$$\mathbf{x}(t_k) = [x(t_k)\ y(t_k)\ z(t_k)\ \dot{x}(t_k)\ \dot{y}(t_k)\ \dot{z}(t_k)\ \ddot{x}(t_k)\ \ddot{y}(t_k)\ \ddot{z}(t_k)\,]^T . \tag{8}$$

In discrete time moment t_k the dynamics of the target's motion can be modeled as a continuous linear, time-invariant system as follows

$$\frac{d\mathbf{x}(t_k)}{dt} = \mathbf{F}\mathbf{x}(t_k) + \mathbf{L}\mathbf{w}(t_k) \tag{9}$$

where $\mathbf{w}(t_k)$ is a white noise process with power spectral density matrix

$$\mathbf{Q}_c = \begin{bmatrix} q & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q \end{bmatrix}, \quad (10)$$

where q is the process noise variance of the acceleration.

The model matrices $\mathbf{F}$ and $\mathbf{L}$ of constant entries, which characterize the behavior of the target are described as follows

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

From the equation (9) it follows that the acceleration of the object is perturbed with a white noise process. Therefore this model has the name continuous Wiener process acceleration (CWPA) model.

The measurement 3-D model matrix $\mathbf{H}$ is set to

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (12)$$

which means that observable are only coordinates of the vector position of the moving UAV. Based on the expressions (4) and (5) for the state transition matrix of the dynamic model $\mathbf{A}_k$ and covariance noise state matrix $\mathbf{Q}_k$ can be written as

$$\mathbf{A}_k = \begin{bmatrix} 1 & 0 & 0 & (\Delta t_k) & 0 & 0 & \frac{(\Delta t_k)^2}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & (\Delta t_k) & 0 & 0 & \frac{(\Delta t_k)^2}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & (\Delta t_k) & 0 & 0 & \frac{(\Delta t_k)^2}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & (\Delta t_k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & (\Delta t_k) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & (\Delta t_k) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$\mathbf{Q}_k = \begin{bmatrix} \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^3}{6} & 0 & 0 \\ 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^3}{6} & 0 \\ 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^3}{6} \\ \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 \\ 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 \\ 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 & \frac{(\Delta t_k)^4}{8} \\ \frac{(\Delta t_k)^3}{6} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 & 0 \\ 0 & \frac{(\Delta t_k)^3}{6} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} & 0 \\ 0 & 0 & \frac{(\Delta t_k)^3}{6} & 0 & 0 & \frac{(\Delta t_k)^4}{8} & 0 & 0 & \frac{(\Delta t_k)^5}{20} \end{bmatrix} q \tag{14}$$

The covariance matrix of the measurements is set to

$$\mathbf{R} = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}, \tag{15}$$

where r_1, r_2, r_3 are the variances of position measurements on x, y and z coordinates.

4. Recurrent Kalman Equations

The main recurrent Kalman equation to calculate the current vector of estimated mean state parameters on k-th moment can be written as

$$\mathbf{m}_k = \mathbf{A}_{k-1}\mathbf{m}_{k-1} + \mathbf{K}_k[\mathbf{y}_k - \mathbf{H}_k(\mathbf{A}_{k-1}\mathbf{m}_{k-1})] \tag{16}$$

where

$$\mathbf{K}_k = (\mathbf{A}_{k-1}\mathbf{P}_{k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1})\mathbf{H}_k^T\mathbf{S}_k^{-1} \tag{17}$$

is the Kalman filter gain on the time moment k;

$$\mathbf{P}_k = \mathbf{A}_{k-1}\mathbf{P}_{k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1} - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T \quad (18)$$

is the current covariance matrix of the state parameters;

$$\mathbf{S}_k = \mathbf{H}_k(\mathbf{A}_{k-1}\mathbf{P}_{k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1})\mathbf{H}_k^T + \mathbf{R}_k \quad (19)$$

is the measurement prediction covariance on the time moment k.
Vector estimate parameters and covariance matrices indexed by (k-1) stand for vector estimate parameters and covariance matrices defined at the previous, (k-1)th moment.
In order to simplify the computational algorithm in programming sense the equation (16) can be decomposed into two systems of equations as follows [16]

Prediction equations:

$$\mathbf{m}_k^p = \mathbf{A}_{k-1}\mathbf{m}_{k-1}$$
$$\mathbf{P}_k^p = \mathbf{A}_{k-1}\mathbf{P}_{k-1}\mathbf{A}_{k-1}^T + \mathbf{Q}_{k-1}$$

where $\mathbf{m}_k^p$ and $\mathbf{P}_k^p$ are the predicted mean value of the state and covariance matrix of the state, respectively, before measurements.
Update equations:

$$\mathbf{m}_k = \mathbf{m}_k^p + \mathbf{K}_k[\mathbf{y}_k - \mathbf{H}_k(\mathbf{A}_{k-1}\mathbf{m}_{k-1})]$$
$$\mathbf{P}_k = \mathbf{P}_k^p - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T$$

where

$$\mathbf{K}_k = \mathbf{P}_k^p\mathbf{H}_k^T\mathbf{S}_k^{-1}$$
$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_k^p\mathbf{H}_k^T + \mathbf{R}_k$$

where $\mathbf{m}_k$ and $\mathbf{P}_k$ are the updated (current) mean value of the state and covariance matrix of the state, respectively, after measurements.

5. Numerical Experiment

In this section a numerical example and simulation results with a linear Kalman filter tracking UAV are presented. The UAV dynamic model starts from origin with zero velocity and application of two-dimensional CWPA acceleration model [16]. The process is simulated in 50 steps.
The entries of the covariance measurement matrix **R** are set as follows r_1 = 10, r_2 = 8, and r_3 = 5. The process noise variance is set to q = 0.2. Initial guesses for the state mean vector and covariance matrix are set to

m = [0 0 0 0 0 0 0 0 0]',

P = Diag [0.1 0.1 0.1 0.1 0.1 0.1 0.5 0.5 0.5].

Noisy measurements are got from the object's position. The velocity is estimated on discrete time steps.

The real position data of the moving object and the simulated position measurements toward both of Cartesian coordinates (*y* and *z*) can be calculated by applying twice 2-D function **gauss_rnd.m**, defined in [16]. It is assumed that the third coordinate of the simulated real and measurement position of the target toward *x* axis is an independent random variable of Gaussian function. Therefore coordinates calculated for the real position and measurement position toward both of axis (*y* and *z*) are completely uncorrelated.

Consider 2-D scenario in (*x, y*) coordinate plane (Fig.1). All results from the simulated numerical experiment depicted in figures are presented in relative dimensions. The real UAV trajectory is depicted by dense line, and measurement UAV trajectory is presented by detached dots. The circle on the lower end of the curve marks the UAV starting point.

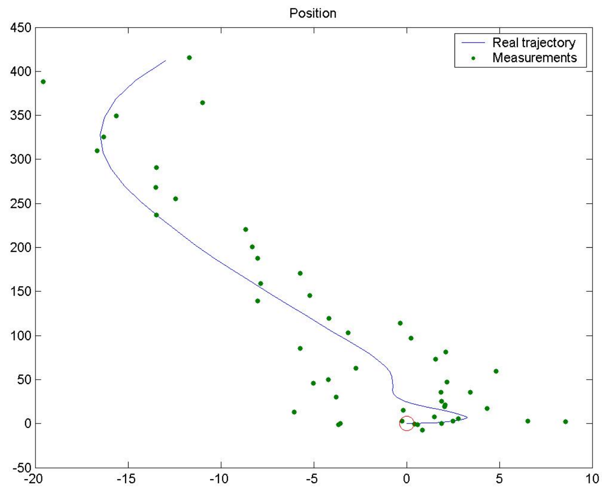
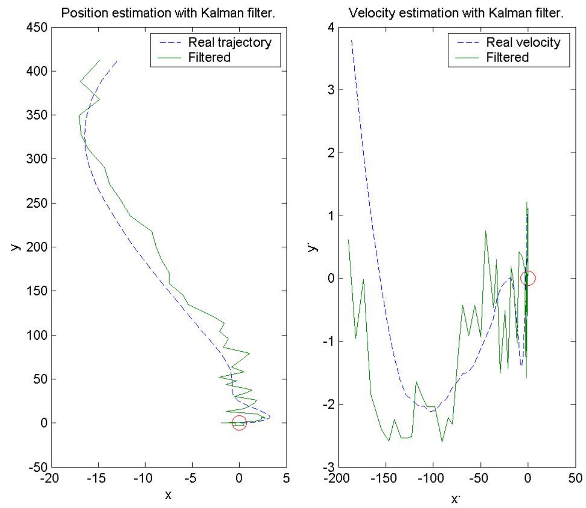


Figure 1. Real trajectory and measurement data of a moving UAV in accordance with CWPA model

Estimates for position and velocity of the moving UAV target using a linear Kalman tracking filter and 2-D CWPA acceleration model in the plane (*x*, *y*) are presented in Fig. 2, *a* and Fig. 2, *b*, respectively. Position estimates are described by real trajectory (dot line) and estimated trajectory (dense line) (*a*). Dynamics of velocity estimates is described by a real velocity line (dot line) and estimated velocity line (dense line) (Fig. 2, *b*).

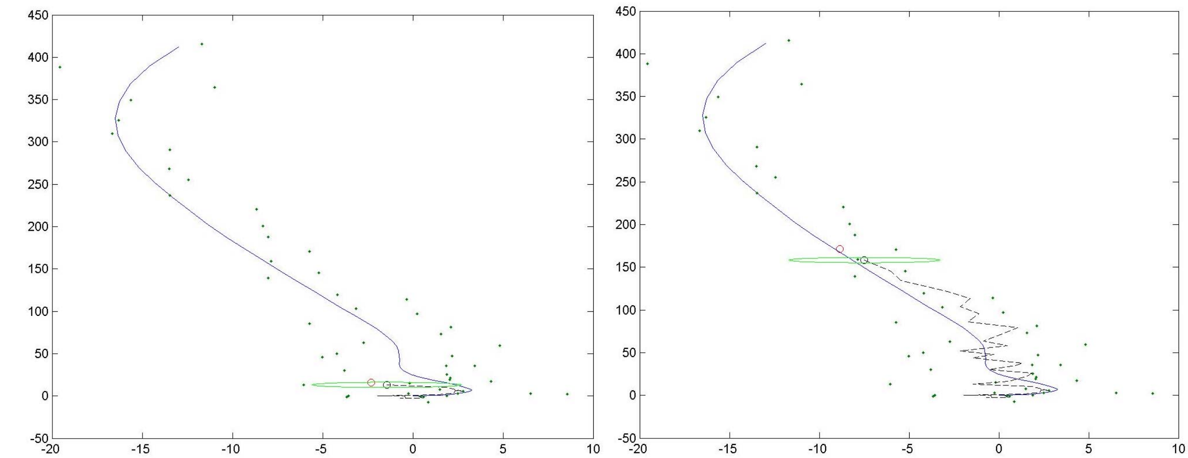


(*a*) (*b*)

Figure. 2. Position estimates: real trajectory (dot line) and estimated trajectory (dense line) (*a*); Velocity estimates: real velocity (dot line) and estimated velocity (dense line) (*b*)

Filtering results can be displayed sequentially on every time step. Real trajectory (solid line) and filtering results (dot line) on 17th and 29th time sequential steps are displayed in Fig. 3, *a* and Fig. 3, *b*, respectively. Real trajectory (solid line) and filtering results (dot line) on 34th and 42nd time sequential steps are displayed in Fig. 4, *a* and Fig. 4, *b*, respectively. Real trajectory (solid line) and filtering results (dot line) on 45th and 49th time sequential steps are displayed in Fig. 5, *a* and Fig. 5, *b*, respectively. The measurement data are displayed as separated dots and the real data simulated by 2-D CWPA model as solid line. The filtering results of previous steps are plotted as dashed line following the real trajectory. The area of a state covariance in each step is displayed as an ellipse around the mean (circle) on each step. The predicted position on next step is displayed as a detached circle.

The results demonstrate satisfactory convergence of the estimated by Kalman filter trajectory to the real one on time sequential steps.



(*a*) (*b*)

Figure. 3. Real trajectory (solid line) and filtering results (dot line) displayed on 17th time sequential step (a) and 29th time sequential step

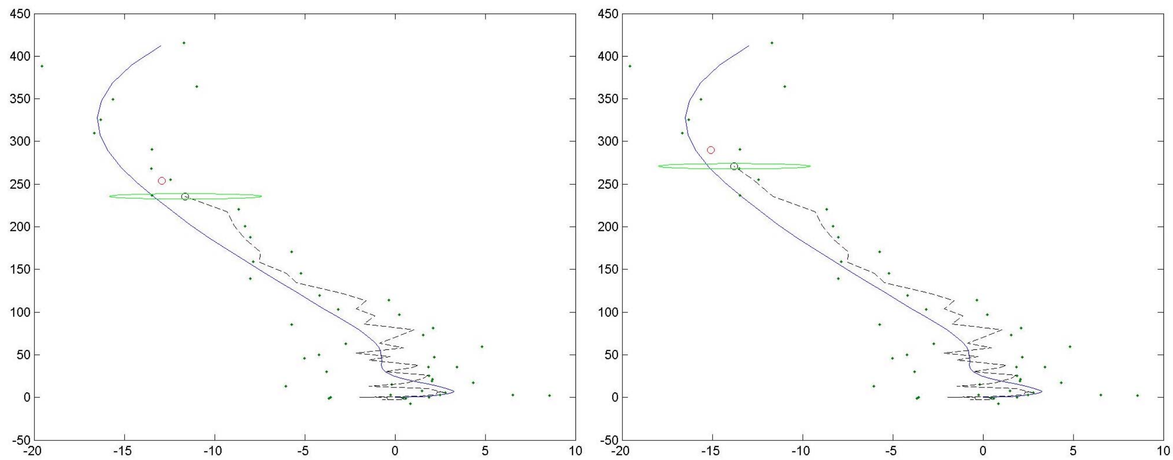


Figure 4. Real trajectory (solid line) and filtering results (dot line) displayed on 34th time sequential step and 42nd time sequential step

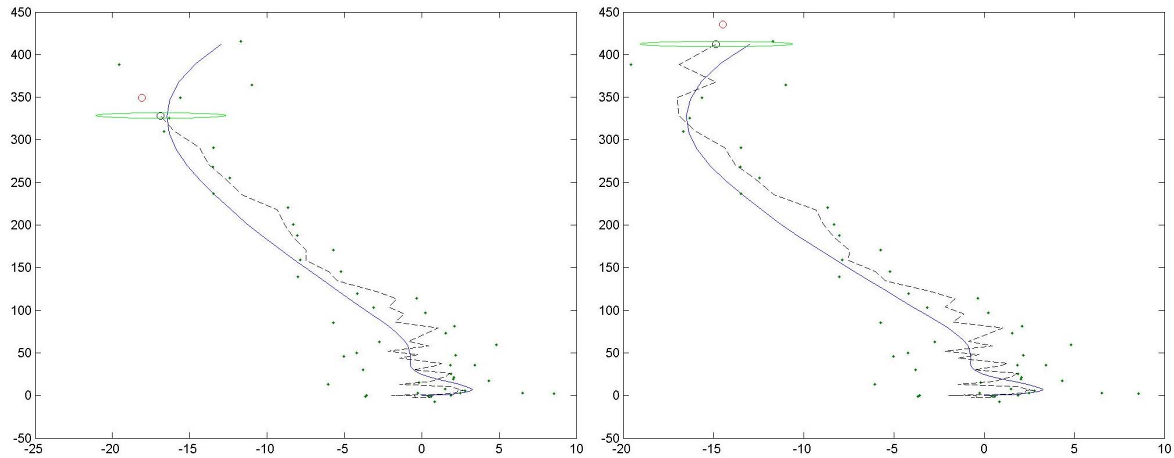


Figure 5. Real trajectory (solid line) and filtering results (dot line) displayed on 45th time sequential step and 49th time sequential step

VI. Conclusion

In the work a Kalman filter for tracking Unmanned Aerial Vehicle (UAV) moving near the base lane of a bistatic radar system and illuminated by GPS signals in three-dimensional (3-D) space have been analyzed and numerically implemented. A three dimensional linear time-invariant dynamic model of the state of a moving UAV has been considered. A linear time-invariant model and Pade expansion of the state transition matrix have been described. A case, where an object is tracked during its movement in 3-D coordinate space with a radar tracking system, is considered. Thoroughly mathematical description of linear time-invariant model matrices has been performed. It includes state matrix vector, power spectral density matrix, model matrices of constant entries, which characterize the behavior of the target, measurement model matrix, dynamic model, and covariance noise state matrix. Modified recurrent matrix equations of a tracking Kalman filter have been presented. Numerical experiment to illustrate the performance of the 3-D dynamics model and the tracking algorithm based on the linear recurrent Kalman filter has been carried out. The results depicted in 2-D coordinate systems show satisfactory convergence of the real and approximated position and velocity estimates. In order to demonstrate a 3-D case of the Kalman tracking procedure it is necessary to be constructed two independent Gaussian functions with different real constants but the same random independent parameter. It is a next step of a future work. The results described in the paper can be used for realization of tracking systems based on measurements of GPS signals reflected by objects moving near the base line of the bistatic forward scattering radar system.

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