GISAR Image Reconstruction Based 2-D Smoothed l0 Norm Minimization in Sparse Decomposition

Andon Lazarov[*], Hristo Kabakchiev[**], Todor Kostadinov[***], Dimitar Minchev[*]

[*]Department Informatics and Technical Sciences, Burgs Free University
No 64 San Stefano Str. 8000 Burgas, BULGARIA
email: lazarov@bfu.bg

[**]Department of Software Technologies, Sofia University “St. Clement of Ohrid”
J. Boucher Blvd. 5, 1164 Sofia, Bulgaria
email: ckabakchiev@fmi.uni-sofia.bg

[***]Department of Technical Sciences, Burgas University As. Zlatarov
Street name, numbers, 00-000 City, BULGARIA
email: kostadinov.todor@yahoo.com

Abstract: *General Inverse Synthetic Aperture Radar (GISAR) imaging algorithm based on two-dimensional (2-D) l0 norm minimization in signal sparse decomposition is discussed. GISAR geometry and kinematics are analytically described. GISAR signal model based on Linear Frequency Modulated (LFM) waveform is derived. The GISAR signal formation process is presented as a sparse decomposition in redundant Fourier basis. Image reconstruction procedure is presented as minimization of smooth norm of the image matrix. The algorithm of minimization of the number of non-zero point scatterers is thoroughly described .The results are illustrated by a numerical experiment.*

1. Introduction

GISAR is a radar imaging technique using movement of the object and radar carrier in order to attain an azimuth resolution. It means that the synthesis of the aperture is performed by displacement of both the object and the radar carrier. The classical approach for GISAR imaging is Range-Doppler signal assessment and compression. It requires large measurement data on both rang and cross-range directions in order to achieve a satisfactory resolution. Nowadays a new approach, Compressed Sensing (CS) based on Sparse Decomposition (SD) in the scope of the SAR analysis and synthesis attracts the attention of many researches. A ground moving target identification method for analysis of off-grid velocities in multichannel SAR based on compressed sensing is suggested in [1]. Improving radar detection performance of CS by orthogonal projection is discussed in [2]. Compressive sensing to remove gaps between frequency channels and improve range resolution in passive ISAR with DVB-T signal is presented in [3]. Compressed sensing application for target elevation estimation via multipath effect in passive radar is illustrated in [4]. Velocity measurements in MTI/MTD radar based on CS is discussed in [5].

The objective of the present work is creation of GISAR CS high precision imaging algorithm based on theory of the sparse decomposition described in the fundamental works [6, 7]. The paper is organized as follows. In section II geometry and kinematics of GISAR scenario is described. In section III transmitted LFM pulses and modeling of GISAR signal deterministic components are analyzed. In section IV sparse decomposition approach to solve the image reconstruction problem is discussed. In section V image reconstruction algorithm based on l0 norm minimization of the GISAR image and sparse decomposition of GISAR signal is described. In section VI results of numerical experiments are presented. In section VII conclusions are made.

2. Geometry and Kinematics of GISAR Scenario

Assume spaceborn SAR illuminates a moving target, a ship on sea. The scenario is depicted in a reference coordinate system of observation $Oxyz$ the origin of which, point O, is placed beneath the radar platform. The target is presented as an assembly of point scatterers placed in the nods of a 3-D regular grid depicted in the coordinate system $O'XYZ$.

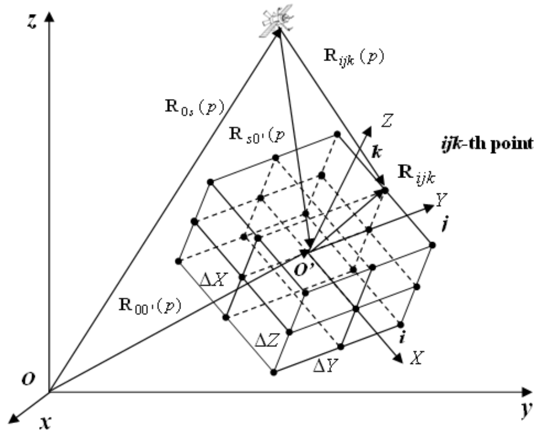

Figure 1. GISAR topology and kinematics

The geometry and kinematics of GISAR scenario is depicted in Fig. 1, where $\mathbf{R}_{0s}(p)$ and $\mathbf{R}_{00'}(p)$ are the current position vectors of the SAR and mass-centre of the target at the pth moment in the rectangular coordinates $Oxyz$, $\mathbf{V}$ are $\mathbf{V}_{0'}$ are the velocity vectors of the SAR carrier and the target, $\mathbf{R}_{ijk}$ is a position vector of ijkth point scatterer in the target area. Based on the geometry the following equations hold: Range distance vector from the SAR to the mass centre of the target

$$\mathbf{R}_{s0'}(p) = \mathbf{R}_{0s}(p) - \mathbf{R}_{00'}(p), \tag{1}$$

where

$$\mathbf{R}_{00'}(p) = \mathbf{R}_{00'}(0) - \mathbf{V}_{0'}\left(\frac{N}{2} - p\right)T_p, \tag{2}$$

is the distance vector measured between coordinates centres, points O and O'; $\mathbf{R}_{00'}(0)$ is the position vector of the target mass centre at the point of imaging, $p = N/2$. The current distance vector of the SAR carrier

$$\mathbf{R}_{0s}(p) = \mathbf{R}_{0s}(0) + \mathbf{V}\left(\frac{N}{2} - p\right)T_p, \tag{3}$$

where T_p is the pulse repetition period, $p = \overline{0, N-1}$, is the current number of emitted pulses, N denotes the total number of emitted pulses during the coherent processing interval.

The current distance vector from the SAR to the ijk-th point scatterer of the target measured at the pth moment

$$\mathbf{R}_{ijk}(p) = \mathbf{R}_{s0'}(p) + \mathbf{A}\mathbf{R}_{ijk}, \tag{4}$$

where $\mathbf{A}$ is the transformation matrix.

3. Transmitted LFM Pulses and Modeling of GISAR Deterministic Signal Components

SAR illuminates a moving target by series of electromagnetic waves, analytically described by sequence of N linear frequency modulated pulses, i.e.

$$\dot{S}(t) = \sum_{p=0}^{N-1} \mathbf{rect}\frac{t_p}{T}\mathbf{exp}\left\{-j\left[\omega t_p + bt_p^2\right]\right\}, \quad (5)$$

where $\mathbf{rect}\frac{t_p}{T} = \begin{cases} 1, if\ 0 \le \frac{t_p}{T} < 1 \\ 0, othewise \end{cases}$

where $t_p = t - pT_p$ is the slow time, $t = pT_p + (k-1)\Delta T$ is the fast time, $k = \overline{0, K-1}$, is the number of LFM pulse sample, K is the full number of LFM samples, $b = 2\pi\Delta F / T$ is the LFM rate, T is the time duration of the LFM pulse, ΔF is the LFM bandwidth.

The deterministic component of the GISAR signal, reflected by *ijk*th point scatterer of the target can be expressed by

$$\dot{S}_{ijk}(\hat{t}) = \sum_{p=1}^{N} a_{ijk}\mathbf{rect}\frac{\hat{t} - t_{ijk}(p)}{T}.\mathbf{exp}\left\{-j\left[\omega\left(\hat{t} - t_{ijk}(p)\right) + b\left(\hat{t} - t_{ijk}(p)\right)^2\right]\right\}. \quad (6)$$

where $\mathbf{rect}\frac{\hat{t} - t_{ijk}(p)}{T} = \begin{cases} 1, 0 \le \frac{\hat{t} - t_{ijk}(p)}{T} < 1, \\ 0, \text{otherwise} \end{cases}$

where a_{ijk} is the reflection coefficient of the ijkth point scatterer, three dimensional (3-D) image function; $t_{ijk\,\min}(p)$ denotes the minimal time delay of the GISAR signal reflected from the nearest point scatterer of the target, $t_{ijk}(p) = \frac{2R_{ijk}(p)}{c}$ is the time delay of the signal from the ijkth point scatterer; $R_{ijk}(p)$ is the distance vector module of the to the point scatterer defined by

$$R_{ijk}(p) = \left[x_{ijk}^2(p) + y_{ijk}^2(p) + z_{ijk}^2(p)\right]^{\frac{1}{2}}, \quad (7)$$

where $x_{ijk}(p), y_{ijk}(p), z_{ijk}(p)$ are the current coordinates of the ijkth point scatterer.

Deterministic complex components of the GISAR signal return from the target are defined as a superposition of signals reflected by all illuminated point scatterers placed on the target, i.e.

$$\dot{S}(\hat{t}) = \sum_{p=1}^{N}\sum_{ijk}\dot{S}_{ijk}(\hat{t}) = \sum_{p=1}^{N}\sum_{ijk} a_{ijk}\mathbf{rect}\frac{\hat{t} - t_{ijk}(p)}{T}\mathbf{exp}\left\{-j\left[\omega\left(\hat{t} - t_{ijk}(p)\right) + \left(\hat{t} - t_{ijk}(p)\right)^2\right]\right\}. \quad (8)$$

Demodulated GISAR signal can be expressed by

$$\tilde{S}(\hat{t}) = \sum_{p=1}^{N}\sum_{ijk} a_{ijk}\mathbf{rect}\frac{\hat{t} - t_{ijk}(p)}{T}\mathbf{exp}\left\{-j\left[\omega(k)\hat{t}_{ijk}(p) + b\hat{t}_{ijk}^2(p)\right]\right\}, \quad (9)$$

where $\omega(k) = \omega + 2b(k-1)\Delta T$ denotes the current angular frequency of the emitted LFM pulse, $\hat{t}_{ijk}(p) = t_{ijk\,\min}(p) - t_{ijk}(p)$. GISAR signal $\tilde{S}(\hat{t})$ is time distributed on two coordinates, range (fast time) and cross range or azimuth (slow time) and can be defined by discrete coordinates p and k as $\tilde{S}(p,k)$, i.e.

$$\tilde{S}(p,k) = \sum_{ijk} a_{ijk}\mathbf{rect}\frac{\hat{t} - t_{ijk}(p)}{T}\mathbf{exp}\left\{-j\left[\omega(k)\hat{t}_{ijk}(p) + b\hat{t}_{ijk}^2(p)\right]\right\}. \quad (10)$$

Taylor expansion $\omega(k)\hat{t}_{ijk}(p)+b\hat{t}_{ijk}^{2}(p)$ in the vicinity of unknown coordinate indices $\hat{p}$ and $\hat{k}$ of the *ijk*th point scatterers yields

$$\widetilde{S}(p,k)=\sum_{\hat{p},\hat{k}}a(\hat{p},\hat{k}).\mathbf{exp}\left\{-j\left[2.\pi.\frac{p.\hat{p}}{\hat{N}}+2.\pi.\frac{k.\hat{k}}{\hat{K}}+\Phi(p,k)\right]\right\}, \tag{11}$$

where $\Phi(p,k)$ is the phase term of higher order, $\hat{p}=\overline{0,\hat{N}-1}$, $\hat{k}=\overline{0,\hat{K}-1}$, $\hat{N}$ and $\hat{K}$ denote the full number of reference image points on cross range and range directions, $a(\hat{p},\hat{k})$ is the 2-D image function. Assume $\Phi(p,k)=0$, then (11) in matrix form can be rewritten as

$$\mathbf{S}=\mathbf{P}.\mathbf{A}.\mathbf{K}^{T} \tag{12}$$

where $\mathbf{S}(N\times K)$ is the measurement signal matrix, $\mathbf{P}(N\times\hat{N})=\left[\exp\left(-j\frac{2\pi p.\hat{p}}{\hat{N}}\right)\right]$ is the Discrete Fourier Transform (DFT) matrix (cross-range matrix-dictionary), $\mathbf{K}(K\times\hat{K})=\left[\exp\left(-j\frac{2\pi k.\hat{k}}{\hat{K}}\right)\right]$ is the DFT matrix (range matrix-dictionary), $\mathbf{A}(\hat{N}\times\hat{K})$ is the image matrix.

4. Sparse Decomposition Approach to Solve the Image Reconstruction Problem

Expression (12) denotes 2-D discrete Fourier decomposition of the signal in matrix form. It means that the two-dimensional signal $\mathbf{S}\in\mathbf{R}^{N\times K}$ is a linear combination of columns of matrices **P** and **K.** In case $N=\hat{N}$ (complete measurement) the decomposition (12) is unique, it means that there exists a unique sparsest solution for **A**. Define compressed (sensed) measurement matrix, $\mathbf{X}=\mathbf{\Phi}_{p}.\mathbf{S}.\mathbf{\Phi}_{k}^{T}+\mathbf{W}\in\mathbf{R}^{N'\times K'}$ over the redundant Fourier dictionaries $\hat{\mathbf{P}}=\mathbf{\Phi}_{p}.\mathbf{P}\in\mathbf{R}^{N'\times\hat{N}}$ and $\hat{\mathbf{K}}=\mathbf{\Phi}_{k}\mathbf{K}.=\mathbf{R}^{K'\times\hat{K}}$, where $\mathbf{\Phi}_{p}(N'\times N)$ and $\mathbf{\Phi}_{k}(K'\times K)$ are pseudo identity sensing matrices**,** $\mathbf{W}$ is the white Gaussian noise matrix. In overcomplete case $N'<\hat{N}$ and $K'<\hat{K}$ the matrix $\mathbf{X}$ does not have unique decomposition. The image reconstruction problem can be solved by definition of sparse decomposition of the measurement signal [3]

$$\mathbf{min}\|\mathbf{A}\|_{0} \text{ subject to } \left\|\mathbf{X}-\hat{\mathbf{P}}.\mathbf{A}.\hat{\mathbf{K}}^{T}\right\|_{2}^{2}\leq\varepsilon, \tag{13}$$

where $\mathbf{min}\|\mathbf{A}\|_{0}$ is the l_0- norm that denotes the number of non-zero point scatterer intensities in image matrix **A**, that means to find out the image matrix **A** with as much zero entries as possible, $\left\|\mathbf{X}-\hat{\mathbf{P}}.\mathbf{A}.\hat{\mathbf{K}}^{T}\right\|_{2}^{2}$ denotes the square of the Euclidian norm, ε is a small constant.

A Gaussian function is used to approximate the l_0- norm, i.e.

$$\|\mathbf{A}\|_{0}=\hat{N}.\hat{K}-\sum_{\hat{p}=0}^{\hat{N}-1}\sum_{\hat{k}=0}^{\hat{K}-1}\exp\left(-\frac{\hat{a}_{\hat{p},\hat{k}}^{2}}{2\sigma^{2}}\right) \tag{14}$$

where σ is the variance of the white Gaussian noise. Then l_0-norm, $\mathbf{min}\|\mathbf{A}\|_0$ can be obtained be maximizing of the Gaussian function $F_\sigma(\mathbf{A}) = \sum_{\hat{p}=0}^{\hat{N}-1}\sum_{\hat{k}=0}^{\hat{K}-1}\exp\left(-\frac{\hat{a}_{\hat{p},\hat{k}}^2}{2\sigma^2}\right)$ onto the feasible set $\left\{\mathbf{A}\middle|\mathbf{X} = \hat{\mathbf{P}}.\mathbf{A}.\hat{\mathbf{K}}^T\right\}$ by a steepest ascent algorithm followed by projection onto the feasible set. Maximization of $F_\sigma(\mathbf{A})$ means increasing the number of zeros entries in the image matrix **A**.

5. Image Reconstruction Algorithm Based on Sparse Decomposition

The modified image reconstruction algorithm is based on the fundamental works [6, 7].

1. Calculate initial estimate of the image matrix $\hat{\mathbf{A}}_0$, using Euclidian norm $\left\|\mathbf{X} - \hat{\mathbf{P}}.\mathbf{A}.\hat{\mathbf{K}}^T\right\|_2^2 = 0$, which corresponds to an initial variance $\sigma_0 = \infty$, i.e.

$$\hat{\mathbf{A}}_0 = \hat{\mathbf{P}}^*.\mathbf{X}.(\hat{\mathbf{K}}^*)^T, \qquad (14)$$

where $\hat{\mathbf{P}}^* = \left[\exp\left(j\frac{2\pi p.\hat{p}}{\hat{N}}\right)\right]$, $\hat{\mathbf{K}}^* = \left[\exp\left(j\frac{2\pi k.\hat{k}}{\hat{K}}\right)\right]$ are the cross range and range inverse DFT matrices, respectively.

2. Define the next value of the variance $\sigma_1 = (2-4).(\max\left|\hat{a}_{\hat{p},\hat{k}}\right|)$ where $(\max\left|\hat{a}_{\hat{p},\hat{k}}\right|)$ is the maximum absolute value of an entry in the matrix $\hat{\mathbf{A}}_0$.

3. Define decreasing sequence of variances $\sigma_j = c.\sigma_{j-1}$, where $j = \overline{2,J}$, $0.5 \le c \le 1$, i.e. For each σ_{j-1}, calculate the $F_\sigma(\mathbf{A}) = \sum_{\hat{p}=0}^{\hat{N}-1}\sum_{\hat{k}=0}^{\hat{K}-1}\exp\left(-\frac{\hat{a}_{\hat{p},\hat{k}}^2}{2\sigma_{j-1}^2}\right)$ and $\|\mathbf{A}\|_0$ by expression (14) .

Steepest ascending algorithm for GISAR imaging

4. Initialization: Let $\mathbf{A} = \hat{\mathbf{A}}_{j-1}$, obtained for $\sigma = \sigma_{j-2}$.

Define the decreasing matrix $\mathbf{\Delta} = [\delta_{\hat{p},\hat{k}}] = [-\sigma_{j-2}^2.(\nabla F_{\sigma j-2})]$, where ∇ is the nabla operator over $F_{\sigma j-2}$, then $\delta_{\hat{p},\hat{k}} = \hat{a}_{\hat{p}.\hat{k}}.\exp\left(-\frac{\hat{a}_{\hat{p},\hat{k}}^2}{2\sigma_{j-2}^2}\right)$.

5. Calculate $\hat{\mathbf{A}}_j = \hat{\mathbf{A}}_{j-1} - \mathbf{\Delta}$. If $\hat{\mathbf{A}}_j < \hat{\mathbf{A}}_{j-1}$, go to step 3. In case the matrix $\hat{\mathbf{A}}_j$ does not change, then project matrix $\hat{\mathbf{A}}_{j-1}$ back onto the feasible set $\left\{\mathbf{A}\middle|\mathbf{X} = \hat{\mathbf{P}}.\mathbf{A}.\hat{\mathbf{K}}^T\right\}$, i.e.

$\mathbf{A}_j = \mathbf{A}_{j-1} - \hat{\mathbf{P}}^*(\hat{\mathbf{P}}.\mathbf{A}_{j-1}.\hat{\mathbf{K}}^T - \mathbf{X})(\hat{\mathbf{K}}^*)^T$.

6. The procedure is repeated until $\mathbf{A}_j = \mathbf{A}_{j-1}$ or $\|\mathbf{A}\|_0$ does not decrease anymore.

6. Numerical Experiment

To illustrate 2-D smoothed l0 norm minimization in GISAR image a numerical experiment is carried out. Assume the target is moving rectilinearly in observation system $Oxyz$ (Fig.1).

Satellite SAR coordinates: $x^s = 250$ m; $y^s = 100$ m; $z^s = 2.10^5$ m; linear velocity 2000 m/s. Target parameters: velocity $V = 18$ m/s, LFM signal parameters: wavelength $\lambda = 3.10^{-2}$ m, PRP $T_p = 2.10^{-3}$ s, pulse width $T = 0,9.10^{-6}$ s, $\Delta T = 3,5.10^{-8}$ s, bandwidth $\Delta F = 1,5.10^8$ Hz. Dimensions of CS matrices: measurement matrix $\mathbf{S}(256 \times 256)$, compressing matrices $\mathbf{\Phi}_p = \mathbf{\Phi}_k(32 \times 256)$, image matrix $\mathbf{A}_0(64 \times 64)$, compressed sensing matrix $\mathbf{X}(32 \times 32)$.

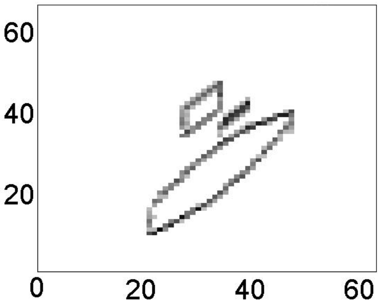

Figure 2. GISAR image after l0 norm minimization

The final image with substantial reduction of noise is depicted in Fig.2. Details of the target are clearly seen. Despite of reduced values of measurements the resolution is satisfactory.

7. Conclusion

GISAR imaging algorithm based on 2-D l0 norm minimization in signal sparse decomposition has been discussed. GISAR geometry, kinematics and LFM signal model have been analytically described. The signal formation process has been presented as a sparse decomposition in redundant Fourier basis. Image reconstruction procedure has been expressed as minimization of smooth norm of the image matrix. Minimization of the number of non-zero point scatterers has been applied in a GISAR image reconstruction procedure.

Acknowledgment

This work is supported by project National Science Fund T02/3, 2014.

References:

[1] L. Prunte. GMTI from multichannel ISAR images using compressed sensing under off-grid conditions, In Proceedings of IRS-2013, Dresden, 19-21 June 2013, pp. 95-100.

[2] Y. Lu, S Hegler, et al. Improving detection performance of compressed sensing by orthogonal projection, In Proceedings of IRS-2013, Dresden, 19-21 June 2013, pp. 107-112.

[3] W.Qiu, E. Giusti, A Bacci, M. Martorella, F. Berizzi et all. Compressive sensing for passive ISAR with DVB-T signal. In Proceedings of IRS-2013, 19-21 June 2013, pp.113-118.

[4] J.S. Kulpa, L. Misiurewicz. Compressed sensing application for target elevation estimation via multipath effect in passive radar. IRS-2013, in the Proceeding of IRS-2013, 19-21 June 2013, pp. 119-124.

[5] J. Misiurewicz. Compressed sensing for velocity measurements in MTI/MTD radar, in the Proceeding of IRS-2013, 19-21 June 2013, pp. 125-130.

[6] A. Ghaffari, M. Babaie-Zadeh, C. Jutter, and H. Moghaddam, "Sparse decomposition of twodimensional signals", Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing, 2009, pp. 3157-3160.

[7] H. Mohimani, M. Babaie-Zadeh, and C. Jutten, "A fast approach for overcomplete sparse decomposition based on smoothed norm," IEEE Trans. on Signal Proc, vol. 57, no. 1, , January 2009, pp. 289–301.